

A note on the buoyancy layer in a rotating stratified fluid

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In a recent paper, Barcilon & Pedlosky (1967), hereafter referred to as B and P, developed the linear theory for steady motions of a rotating, stratified fluid. The discussion was confined to flows within a circular, cylindrical container which rotated about its axis of symmetry. In §4 it was suggested that for $E^{\frac{3}{2}} < \sigma S < E^{\frac{1}{2}}$ (using the notation of B and P) the boundary layer at the side walls had a triple structure and included a buoyancy layer of thickness $E^{\frac{1}{2}}(\sigma S)^{-\frac{1}{2}}$. According to the calculations in §5, the coefficient governing the magnitude of the buoyancy layer vanishes, so that to first order this layer is absent. We maintain that this layer, which is the innermost layer, is required to satisfy the side wall boundary conditions on both the heat flux and the vertical velocity and that its omission was due to an arithmetic error.

Equation (5.5*b*) of B and P which represents the boundary condition at $r = a$ on the heat flux is $\bar{T}_\xi + \hat{T}_\eta + \tilde{T}_\rho = \frac{1}{4}(v_T(a) - v_B(a))$.

This leads to the corrected form of (5.14) as

$$-\frac{1}{2}(z - \frac{1}{2})[v_T(a) + v_B(a)] - \sum_{n=1}^{\infty} (n\pi)^{-1} [(-1)^n v_T(a) + v_B(a)] \sin n\pi z - \frac{A(z)}{\sqrt{2}} = \frac{1}{4}[v_T(a) - v_B(a)],$$

which together with the Fourier series given in B and P implies that

$$A(z) = \frac{v_B(a) - v_T(a)}{\sqrt{2}}.$$

Returning to the buoyancy layer solution (4.18) we see that

$$\begin{aligned} \tilde{u} &= \tilde{v} = 0, \\ \tilde{w} &= \frac{v_B(a) - v_T(a)}{\sqrt{2}} e^{-\rho/\sqrt{2}} \sin \frac{\rho}{\sqrt{2}}, \\ \tilde{T} &= \frac{v_B(a) - v_T(a)}{\sqrt{2}} e^{-\rho/\sqrt{2}} \cos \frac{\rho}{\sqrt{2}}. \end{aligned}$$

The solution is *independent of* z and is required to satisfy the boundary condition (5.5*b*) on the heat flux. Associated with this horizontal temperature variation is the vertical velocity \tilde{w} which is zero at the side walls. As the smaller vertical velocities in the other layers do not vanish at $r = a$, it is essential that the buoyancy layer does exist.

REFERENCE

- BARCILON, V. & PEDLOSKY, J. 1967 *J. Fluid Mech.* **29**, 609.